A Theory of Stable Market Segmentations

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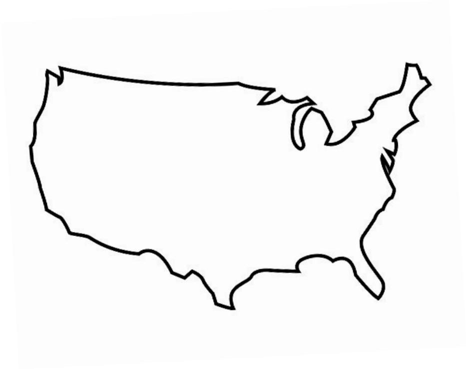
joint with Ron Siegel (Penn State)

February 9, 2023

Market Segmentation



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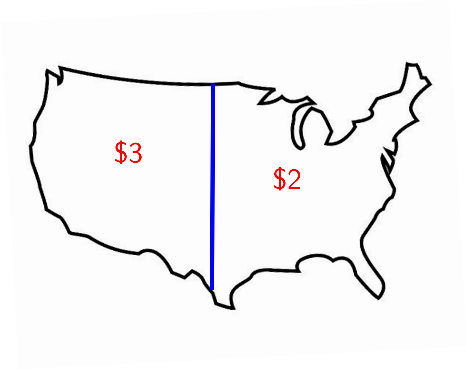
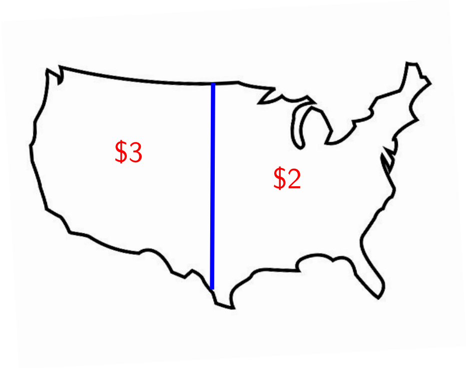


Where do segmentations come from?



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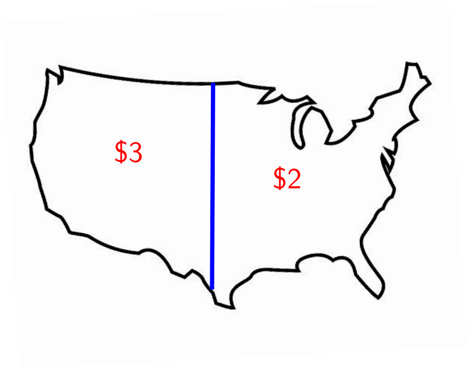

If consumers choose?

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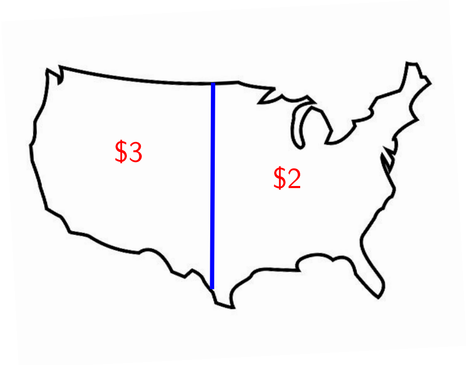

If consumers choose?

- Segment = a coalition of consumers

Platforms, clubs, cooperatives

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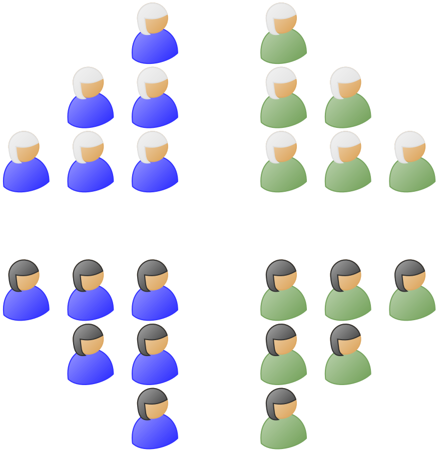

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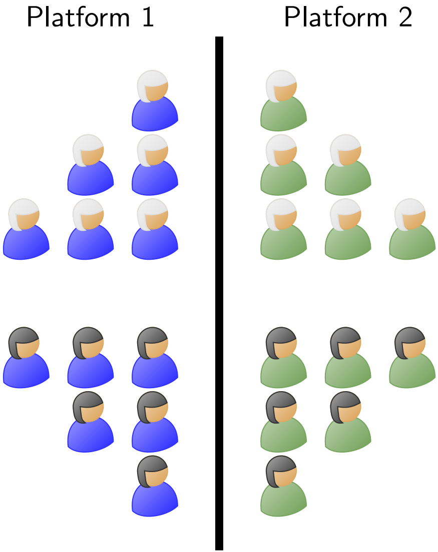

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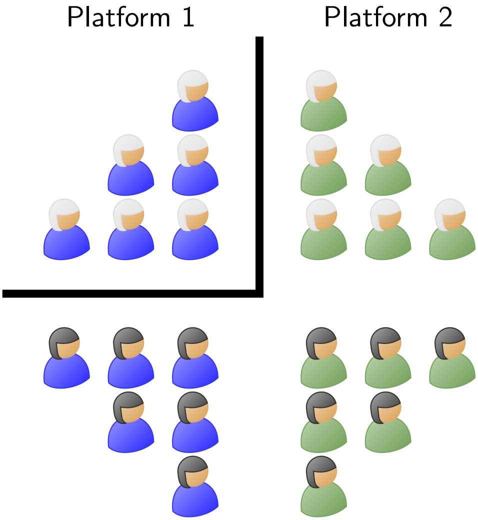

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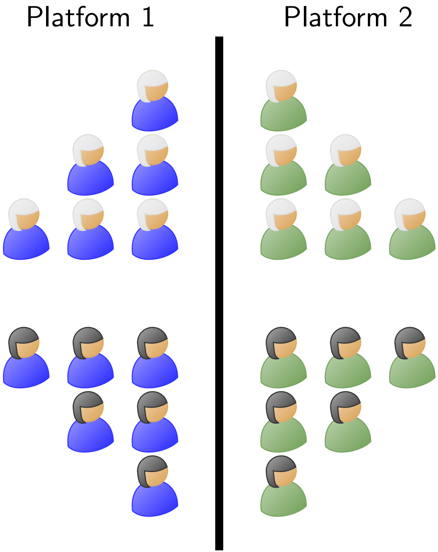

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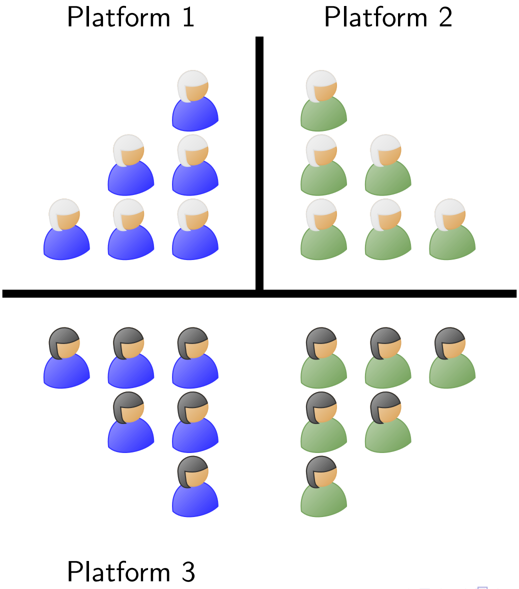

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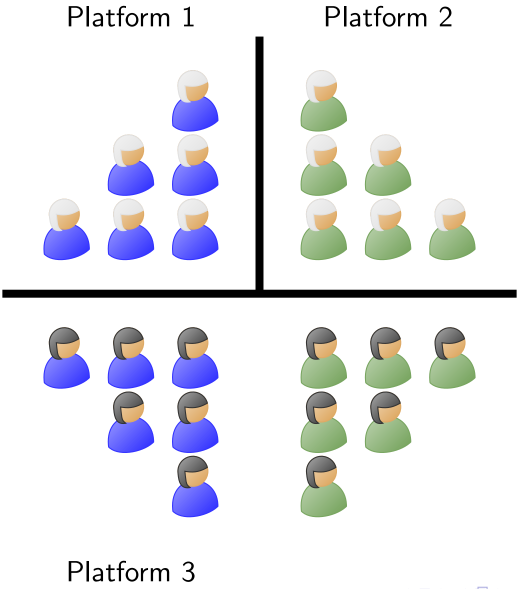

coops

“Stable” segmentations

"Stable" segmentations have "good welfare properties"

Coalitions, segments, and segmentations

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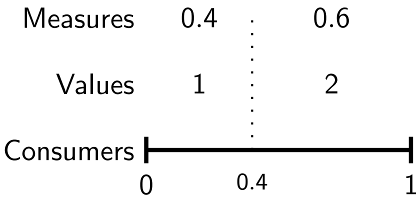

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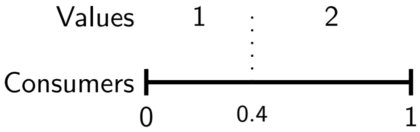

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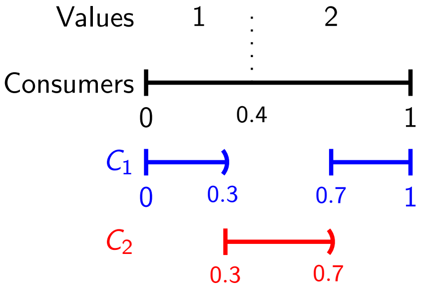

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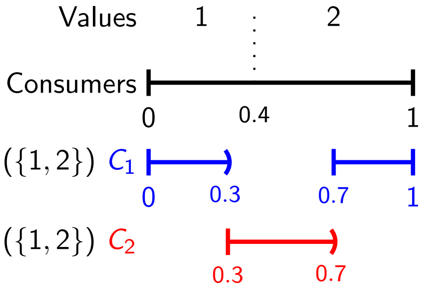

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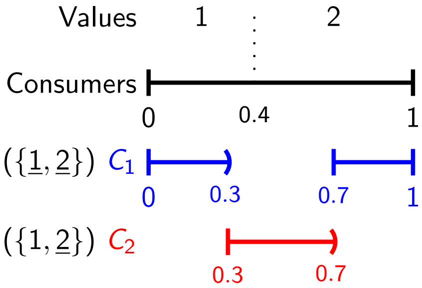

Coalitions, segments, and segmentations

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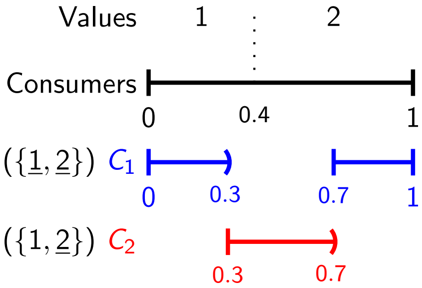

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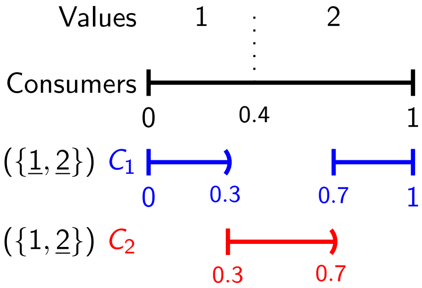

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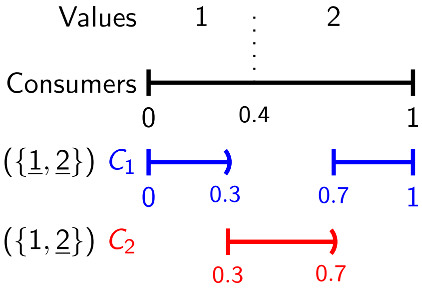
Segmentation $S = \{(C_1, 1), (C_2, 2)\}$ s.t. coalitions partition $[0, 1]$

- $\forall c \in C_1$, $CS(c, S) = \max\{v(c) - 1, 0\}$
- $\forall c \in C_2$, $CS(c, S) = \max\{v(c) - 2, 0\}$



Outline

1. Core
2. Stability

The core

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Definition (Objection)

A segment (C, p) objects to segmentation S if

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Definition (Core)

S is in the core if $\nexists$ segment (C, p) that objects to S

Core is empty in non-trivial cases

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Let v_1 be the lowest possible value

Proposition

1. *If price v_1 is revenue-maximizing to sell to $[0, 1]$,*
2. *If price v_1 is not revenue-maximizing to sell to $[0, 1]$,*

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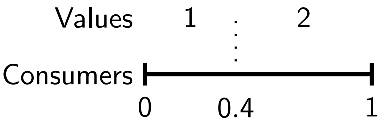
$$\textit{Core is empty}$$

Essentially unique: If S' in core, then $S' \approx \{([0,1], v_1)\}$

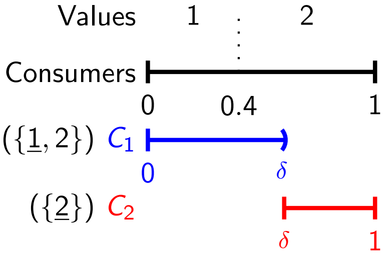
- $S' \approx S$: $CS(c, S') = CS(c, S)$ for (almost) all $c \in [0,1]$

Two type illustration



Two type illustration

If $\delta < 0.8$: $S = \{(C_1, 1), (C_2, 2)\}$ not in core



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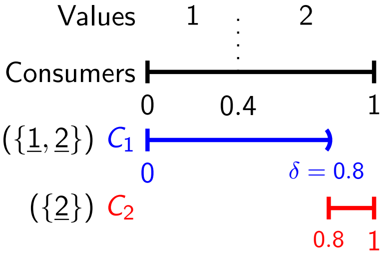

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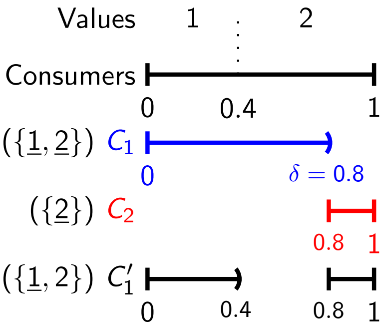

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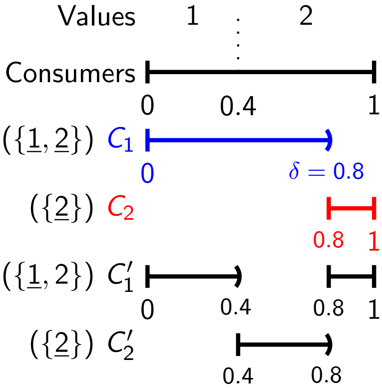

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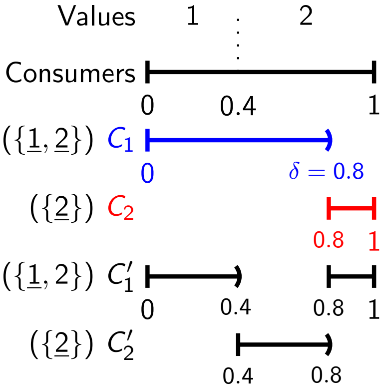
- Segment $(C_1', 1)$ objects
- But $(C_1, 1) \in S$ <u>also objects</u> to resulting $S' = \{(C_1', 1), (C_2', 2)\}$



Stability

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Existing coalitions have sovereignty.

Two type illustration and stability

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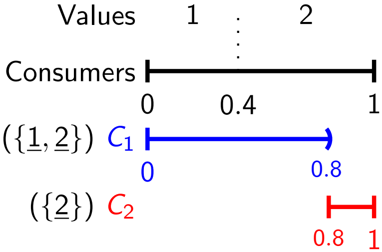
- $(C_1, 1)$ objects to any $S' \not\approx S$



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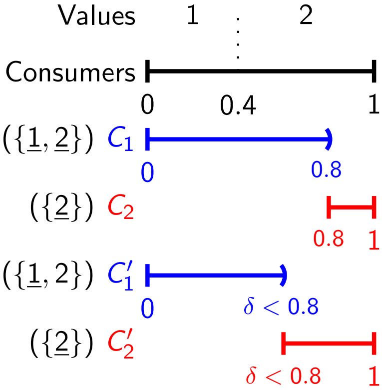
- $(C_1, 1)$ objects to any $S' \not\approx S$

$S' = \{(C_1', 1), (C_2', 2)\}$ is not stable

- S objects to S' but S' doesn't object to S



Comparing solution concepts

Definition (Stability)

S is stable if there is no deviation from it

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Objection in S' has the power to force a move

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Objection in S has the power to prevent a move

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Characterization of stable segmentations

Proposition

1. *Segmentation is stable iff its induced canonical segmentation is stable*
2. *Canonical segmentation S is stable iff it is efficient and saturated*

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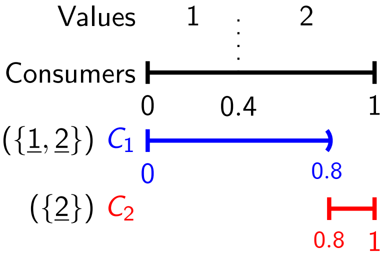
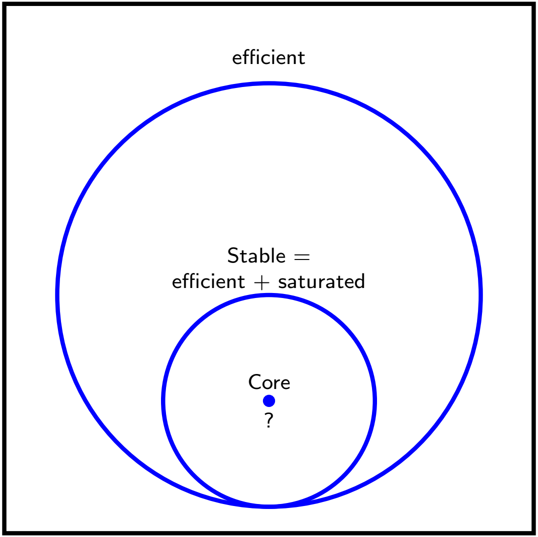


Segmentations



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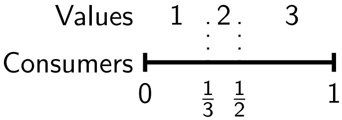

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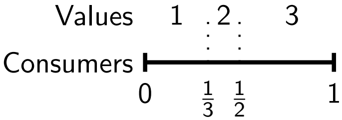

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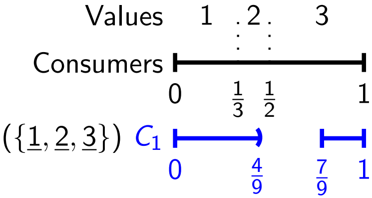

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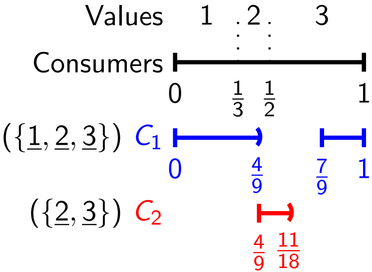

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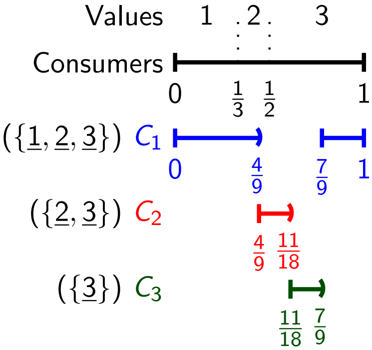

Maximal equal-revenue (MER) segmentation

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Is defined recursively. Let $\bar{C} = [0, 1]$, $S = \emptyset$

1. $C :=$ largest coalition where all prices (among remaining values in $\bar{C}$) are revenue-maximizing
2. Add $(C, \underline{v}(C))$ to S
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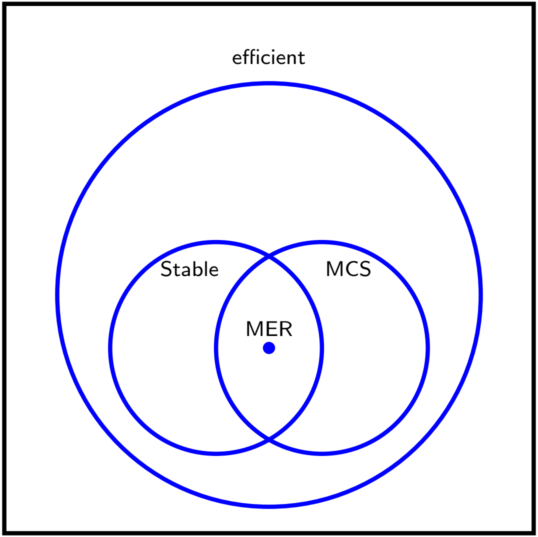
Bergemann, Brooks, Morris (2015):

- The MER segmentation maximizes consumer surplus
- But is not the only one

Segmentations



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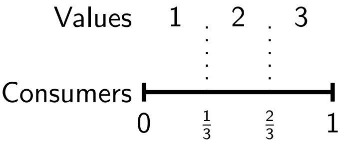
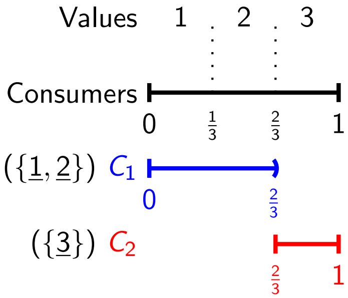


Stability $\not\Rightarrow$ maximizing consumer surplus

$S = \{(C_1, 1), (C_2, 3)\}$ is efficient and saturated $\Rightarrow$ stable



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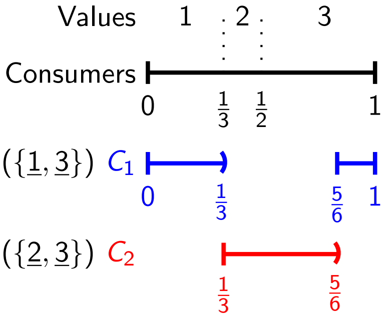

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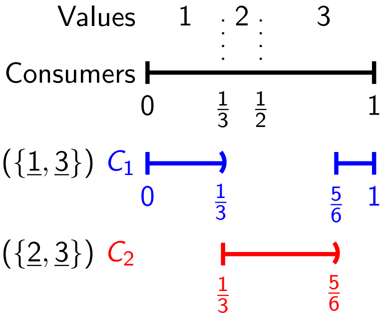
- Efficient allocation
- price 3 is revenue-maximizing for $C_1, C_2, [0, 1]$



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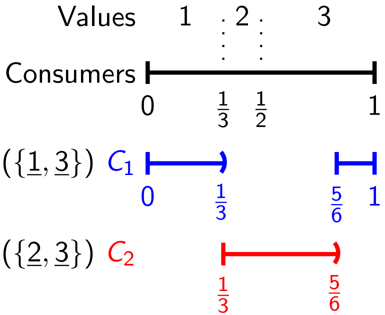
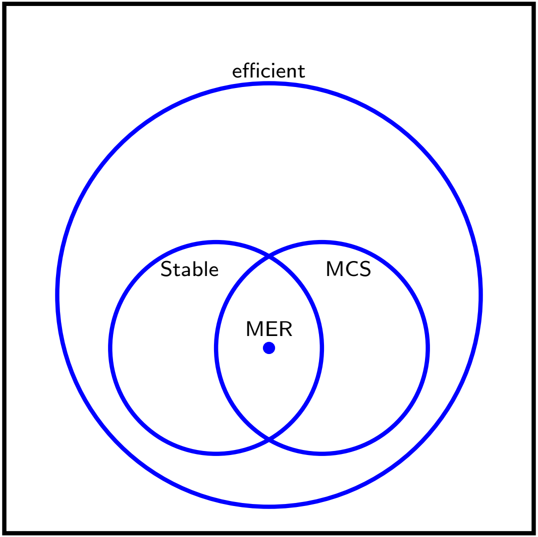
S is not saturated and so not stable:



Segmentations



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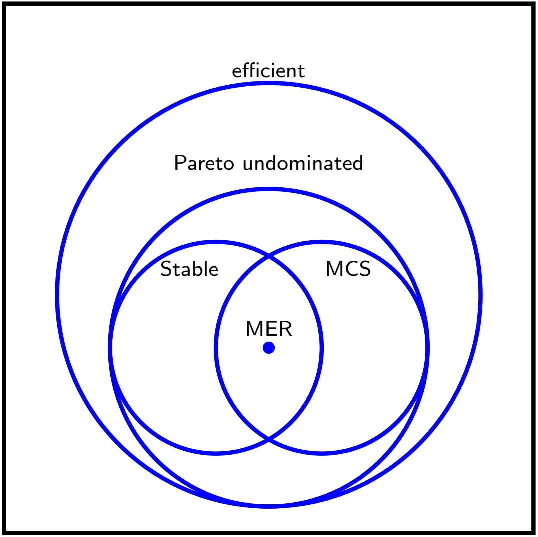

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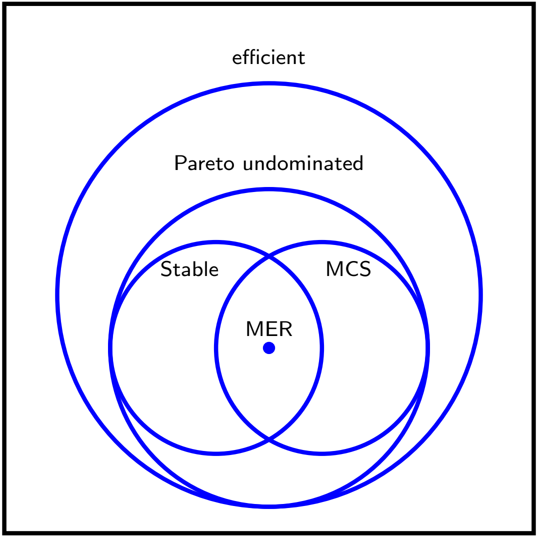
\end{aligned}
$$

Proposition

Stable $\subset$ *Pareto undominated* $\subset$ *efficient*

Segmentations



Related work

Markets as coalitional games

- Shapley (1959); Shubik (1959); . . .; Peivandi and Vohra (2021)
- Core vs. CE: Edgeworth (1881); Debreu and Scarf (1963)

Third degree price discrimination

- Pigou (1920); Robinson (1969); Schmalensee (1981); Varian (1985); Aguirre, Cowan, Vickers (2010); Cowan (2016); . . .

Decentralized Exchanges

- Malamud and Rostek (2017); Chen and Duffie (2021)

Information design

- All segmentations: Bergemann, Brooks, Morris (2015)
- Maximize CS: Hidir and Vellodi (2018); Ichihashi (2020)

Other solutions concepts

- Stable sets (vNM, Harsanyi, Ray and Vohra) details
- Bargaining set details

Problem: market power leads to inefficiency

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Tools:

1. Antitrust
2. Regulated natural monopolist

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3. This paper: market segmentation

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   - "Perfect" segmentation: efficient, eliminates consumer surplus

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How to implement stable segmentations?

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How to implement stable segmentations?

- Ensure coalitional sovereignty

Consumer's control over their data

The Commission recognizes the need for flexibility to permit [...] uses of data that benefit consumers.

("Consumer Privacy in an Era of Rapid Change", FTC, 2012)

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- Data cooperatives

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back

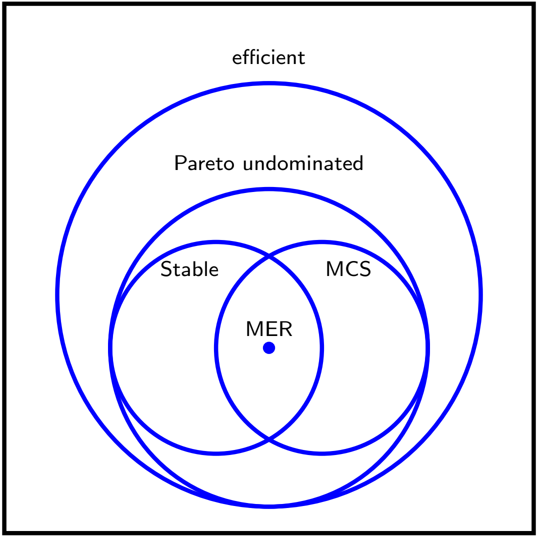
Conclusions

Market segmentation as a tool for achieving efficiency

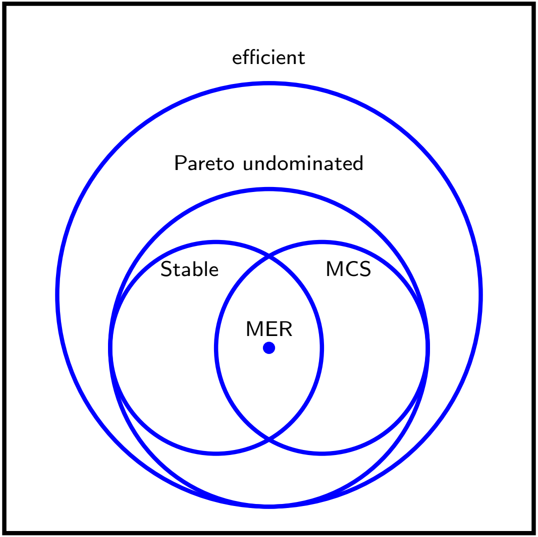
Market segmentation subject to "coalitional sovereignty"

- Stable segmentations are efficient and saturated
  - They are all Pareto un-dominated
  - One of them maximizes consumer surplus

Segmentations



Segmentations



Thanks!

Recall: Stability

Definition

S is stable if it objects to any $S' \not\approx S$

Stable set (von Neumann and Morgenstern 1944)

Definition

A set of segmentations $\mathcal{S}$ is a stable set if

Stable set (von Neumann and Morgenstern 1944)

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A set of segmentations $\mathcal{S}$ is a stable set if

1. Internal Stability: $\forall S \in \mathcal{S}$, $\nexists S' \in \mathcal{S}$ that objects to S
2. External Stability: $\forall S \notin \mathcal{S}$, $\exists S' \in \mathcal{S}$ that objects to S

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If S is stable then $\{S' : S' \approx S\}$ is a stable set:

- $S' \approx S$ doesn't object to S
- S objects to any $S' \not\approx S$

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If S is stable then $\{S' : S' \approx S\}$ is a stable set:

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- S objects to any $S' \not\approx S$

Proposition

$\mathcal{S}$ is stable set iff $\mathcal{S} = \{S' : S' \approx S\}$, s.t. S weakly objects to any $S'' \not\approx S$.

Other stable sets

Definition

- S Harsanyi-objects to S' if exists $S' = S^0, S^1 \ni C^1, \dots, S^k = S \ni C^k$ s.t. $CS(c, S^{i-1}) \leq CS(c, S)$ for all $c \in C^i$ ($<$ for some).
- S Ray-Vohra-objects to S' if exists $S' = S^0, S^1 \ni C^1, \dots, S^k = S \ni C^k$ s.t. $CS(c, S^{i-1}) \leq CS(c, S)$ for all $c \in C^i$ ($<$ for some), and $C \in S^i$ if $C \in S^{i-1}$ and $C^i \cap C = \emptyset$.

back

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Proposition

The following are equivalent for any set of segmentations $\mathcal{S}$:

- *$\mathcal{S}$ is a Harsanyi stable set*
- *$\mathcal{S}$ is a RV stable set*
- *$\mathcal{S} = \{S' : S' \approx S\}$ where S is Pareto undominated.*

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Other solution concepts: Bargaining set

For each objection, $\exists$ stronger objection to same segmentation

$$S'' \xleftarrow{(C', p')} S \xrightarrow{(C,p)} S'$$

back

Other solution concepts: Bargaining set

For each objection, $\exists$ stronger objection to same segmentation

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Stability: for each objection, $\exists$ objection to resulting segmentation

$$S \underset{(C, p) \in S}{\overset{(C', p') \in S'}{\rightleftarrows}} S'$$

back

Other solution concepts: Bargaining set

For each objection, $\exists$ stronger objection to same segmentation

$$S'' \xleftarrow{(C', p')} S \xrightarrow{\text{(C,p)}} S'$$

Stability: for each objection, $\exists$ objection to resulting segmentation

$$\begin{tikzcd} S \arrow[r, shift left, "{(C', p') \in S'}"] & S' \arrow[l, shift left, "{(C, p) \in S}"] \end{tikzcd}$$

Any segmentation is in the bargaining set

back

Other solution concepts: Bargaining set

For each objection, $\exists$ stronger objection to same segmentation

$$S'' \xleftarrow{(C',p')} S \xrightarrow{\text{(C,p)}} S'$$

Formally: $\forall$ objection $(C,p), \exists$ counter-objection (C',p'):

- $CS(c,(C',p')) \geq CS(c,S)$ for all $c \in C' \setminus C$
- $CS(c,(C',p')) \geq CS(c,(C,p))$ for all $c \in C' \cap C$

Stability: for each objection, $\exists$ objection to resulting segmentation

$$S \underset{(C,p)\in S}{\overset{(C',p')\in S'}{\rightleftarrows}} S'$$

Any segmentation is in the bargaining set

back

Other solution concepts

kernel, nucleolus

- Similar to bargaining set
- Not applicable to NTU games
  - need to measure “dissatisfaction” of coalitions

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