Good Data and Bad Data: The Welfare Effects of Price Discrimination

Maryam Farboodi (MIT Sloan), Nima Haghpanah (Penn State), Ali Shourideh (CMU)

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(Third degree) Price discrimination

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- Pigou (1920): true for linear demands

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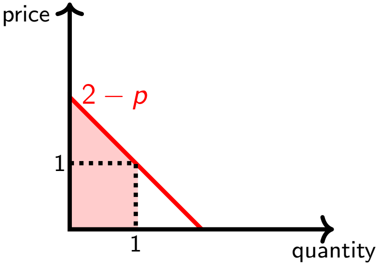

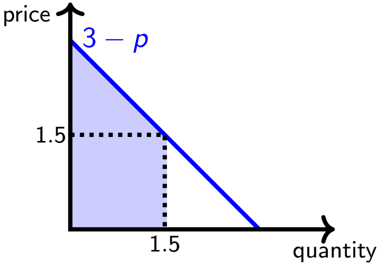

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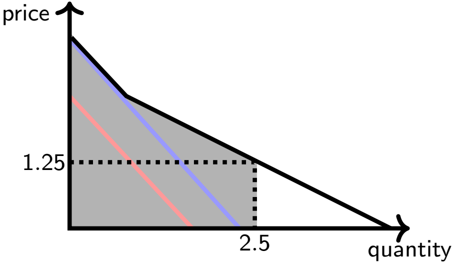

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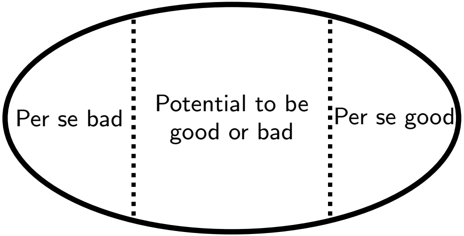

Extended Pigou logic: Linear demands ⇒ Per se bad for TS-CS

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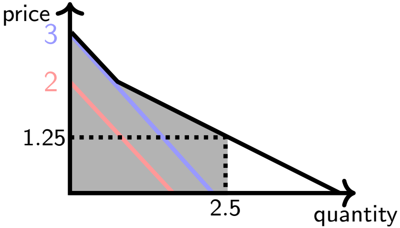

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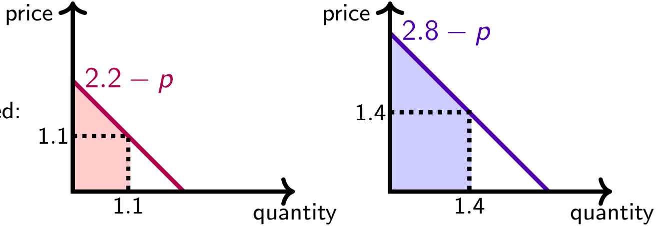

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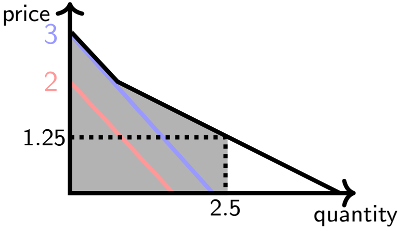

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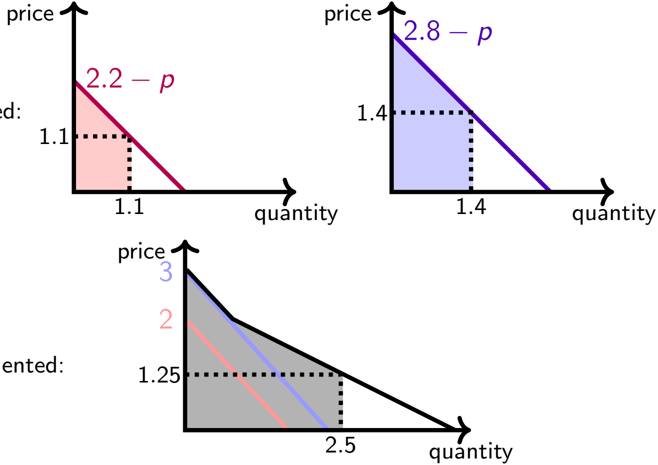

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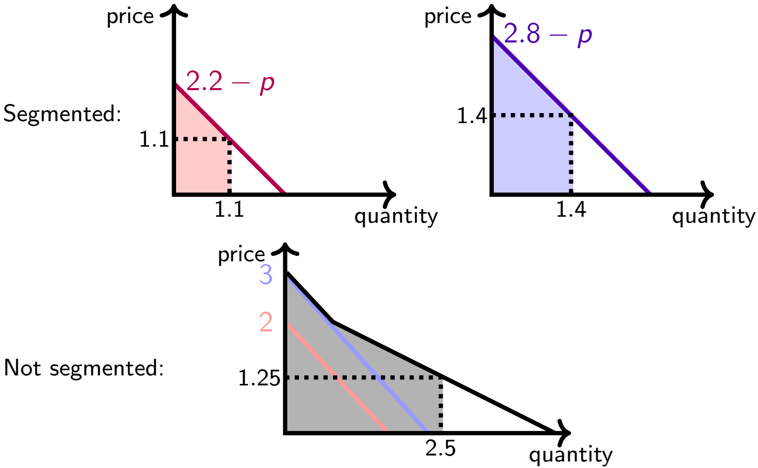

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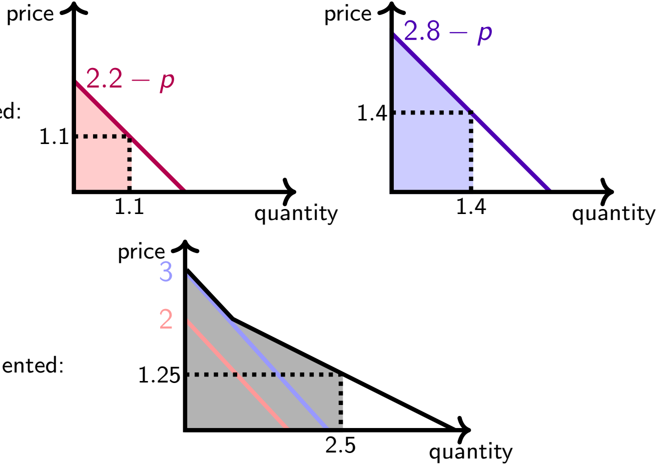

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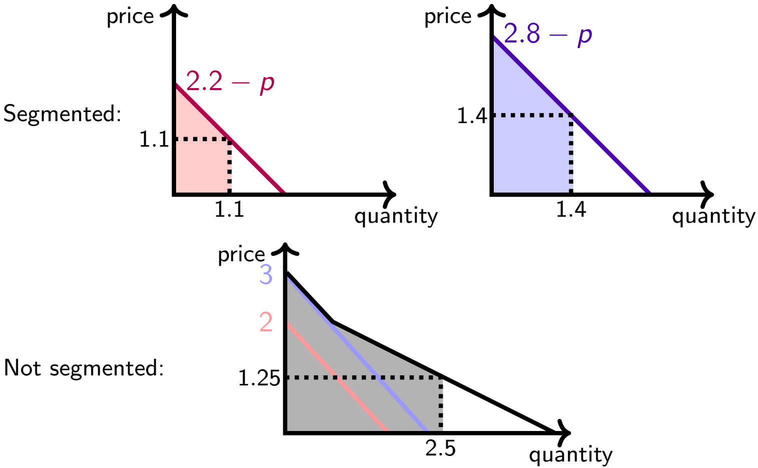
2. Sells that output at different prices: "misallocation effect" < 0



Can information be per se good for CS?

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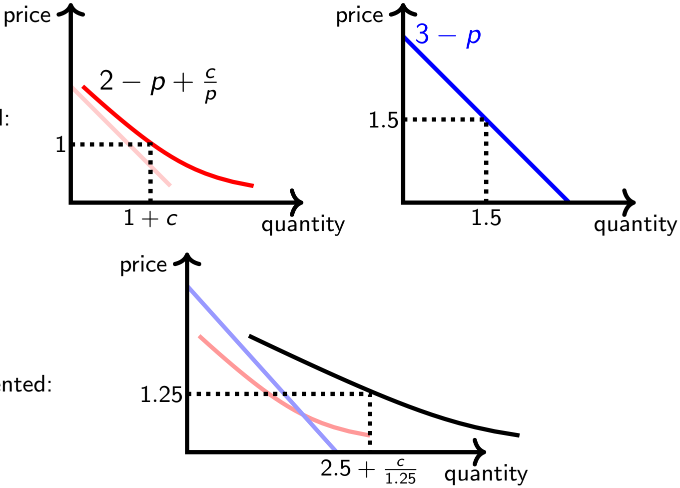

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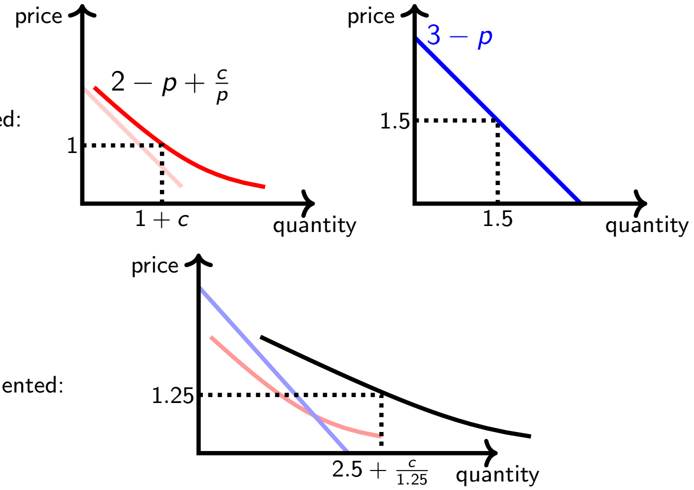

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$c \uparrow \Rightarrow$ "weak" market "level" $\uparrow \Rightarrow$ benefit of PD $\uparrow$

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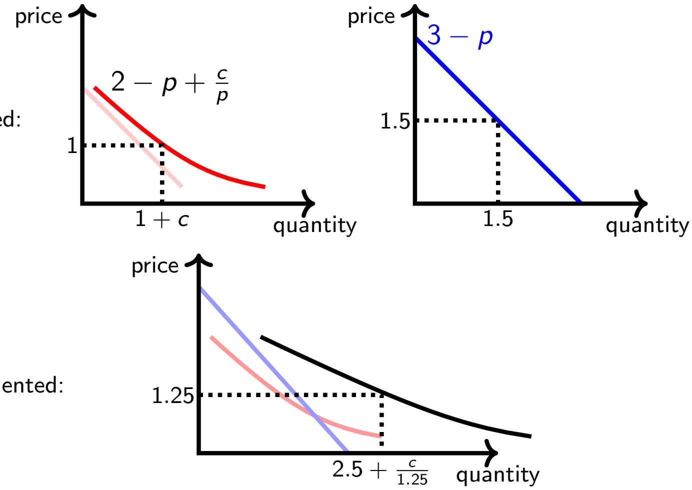

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Split $(\frac{1}{2}, \frac{1}{2})$ into $(\frac{2}{3}, \frac{1}{3})$ and $(\frac{1}{3}, \frac{2}{3})$ with prices $p_1 < p_2$:

The three effects of information

		p_1	p	p_2
Before	Type 1	0	1	0
	Type 2	0	1	0
After	Type 1	$\frac{2}{3}$	0	$\frac{1}{3}$
	Type 2	$\frac{1}{3}$	0	$\frac{2}{3}$

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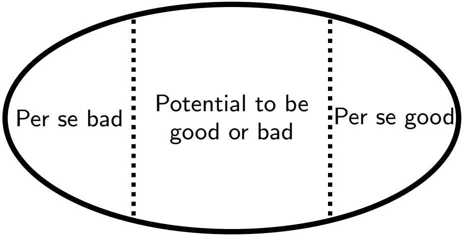
1. The within-type price change effect
2. The cross-types price change effect
3. The price curvature effect

Results

A characterization of:

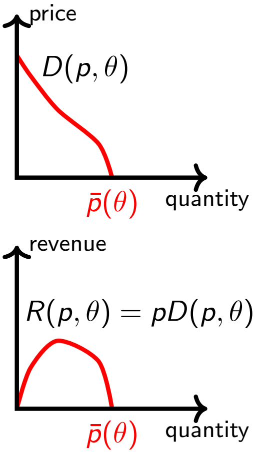


1. A reduction of the problem to one where there is only two types
2. A formula for the two-type case
   - captures the three effects of information

Model

A set of demand curves $\mathcal{D} = \{D(p, \theta)\}_{\theta \in \Theta}$, a prior $\mu_0 \in \Delta(\Theta)$.

- $D(\cdot, \theta)$ downward sloping with concave revenue over $[0, \bar{p}(\theta)]$



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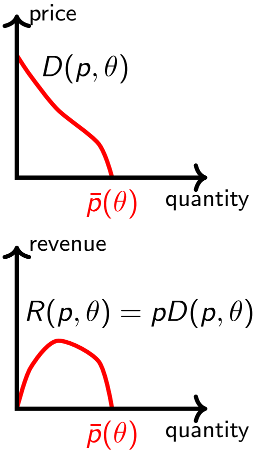
A segmentation: distribution $s \in \Delta(\Delta(\Theta))$ over "markets" $\mu \in \Delta(\Theta)$.

- s.t. $E_{\mu \sim s}[\mu] = \mu_0$.



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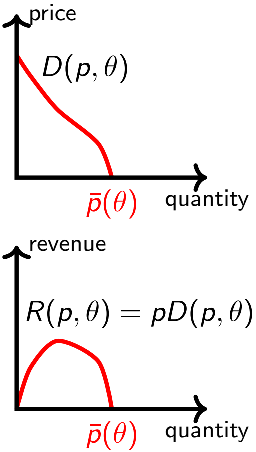
- Seller chooses a price for every market $\mu \in \text{supp}(s)$: $p^*(\mu) \in \arg\max_p \mathbb{E}_{\theta\sim\mu}[R(p,\theta)]$.
- Leads to (weighted) surplus $V^\alpha(s) = \mathbb{E}_{\mu,\theta}[\alpha CS(p^*(\mu),\theta) + (1-\alpha)R(p^*(\mu),\theta)]$



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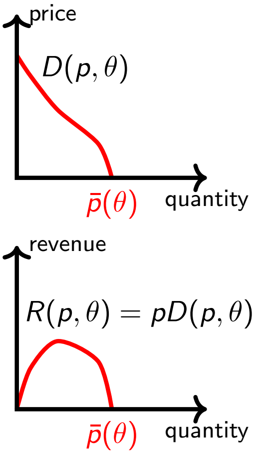
"Information is monotonically α-bad" (α-IMB) if $\forall s, s'$

if "s is finer than s'": s is a mean-preserving spread of s'

$\Rightarrow$ s gives a lower (α-weighted) surplus: $V^\alpha(s) \leq V^\alpha(s')$



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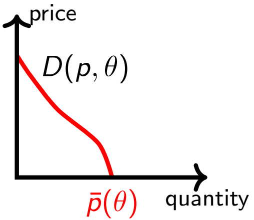
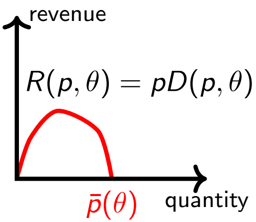




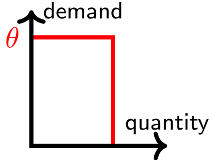
Bridging the classic vs. modern approaches

Classic literature (Pigou 1920, Robinson 1933, Varian 1985, Aguirre et al 2010): same primitives $(\mathcal{D}, \mu)$

- Compare only perfect segmentation and no segmentation

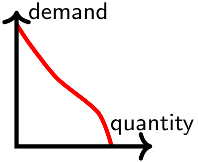
Modern literature (BBM): a family of unit-demand curves

- Values can be perfectly learned
- They ask different questions



We separate types from values

- A type is what's maximally learnable
- Consumers of one type still have heterogeneous values
- First-degree price discrimination is impossible



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Theorem

Let D_1, D_2 *be demands in* $\{D(p,\theta)\}_\theta$ *with lowest and highest monopoly price* p_1^*, p_2^*.

1. α*-IMB (*α*-IMG) holds for* $\{D(p,\theta)\}_\theta$ *if and only if*
   a. *there is no exclusion:* $p^*(\theta) \leq \bar{p}(\theta')$ *for all* $\theta, \theta' \in \Theta$
   b. *there exist two functions* $f_1, f_2 : \Theta \to R_+ \geq 0$ *such that*

$$D(p,\theta) = f_1(\theta)D_1(p) + f_2(\theta)D_2(p), \forall \theta, p \in (p_1^*, p_2^*)$$

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   $$V_2(p) - V_1(p) + \frac{-\frac{R_1'(p)}{R_2'(p)}V_2' + V_1'}{-\frac{R_1'(p)}{R_2'(p)}R_2'' + R_1''}(R_1'(p) - R_2'(p))$$

   is decreasing (increasing) on (p_1^*, p_2^*).

$V_i(p) = V_i^\alpha(p) = \alpha CS_i(p) + (1-\alpha)R_i(p)$.

Outline

1. Implications
2. Intuitions
3. Examples and applications

Implication 1: If α-IMB $\Rightarrow$ α'-IMB $\forall \alpha' > \alpha$

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&= \alpha \left[CS_2(p) - CS_1(p) + \frac{-\frac{R_1'(p)}{R_2'(p)} CS_2' + CS_1'}{-\frac{R_1'(p)}{R_2'(p)} R_2'' + R_1''} (R_1'(p) - R_2'(p)) \right] \\
&+ (1-\alpha) \left[R_2(p) - R_1(p) + \cancelto{0}{\frac{-\frac{R_1'(p)}{R_2'(p)} R_2' + R_1'}{-\frac{R_1'(p)}{R_2'(p)} R_2'' + R_1''} (R_1'(p) - R_2'(p))} \right].
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& + (1-\alpha) \Bigg[R_2(p) - R_1(p) + \cancelto{0}{\frac{-\frac{R_1'(p)}{R_2'(p)} R_2' + R_1'}{-\frac{R_1'(p)}{R_2'(p)} R_2'' + R_1''} (R_1'(p) - R_2'(p))} \Bigg].
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$$

Second expression is increasing over $(p_1^*, p_2^*) : R_1'(p) < 0 < R_2'(p)$.

- If the convex combination is decreasing $\Rightarrow$ decreasing for $\alpha' > \alpha$.

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Corollary

Consider $\{D_1, D_\epsilon\}$ *such that* $\lim_{\epsilon \to 0} p_\epsilon^* = p_1^*$. *There exists* $\hat{\epsilon}, \hat{\alpha}$ *such that for all* $\epsilon < \hat{\epsilon}$,

1. *For all* $\alpha < \hat{\alpha}$, α*-IMG holds for* $\{D_1, D_\epsilon\}$.
2. *For all* $\alpha > \hat{\alpha}$, α*-IMB holds for* $\{D_1, D_\epsilon\}$.

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Example (CES demands)

Consider two demand curves $(c+p)^{-\theta_1}, (c+p)^{-\theta_2}$ for $\theta_1 > \theta_2 > 1$ and some constant $c > 0$. Then $\frac{1}{2}$-IMB holds if and only if $\theta_1 \leq \theta_2 + \frac{1}{2}$.

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Example (CES demands)

Consider two demand curves $(c+p)^{-\theta_1}, (c+p)^{-\theta_2}$ for $\theta_1 > \theta_2 > 1$ and some constant $c > 0$. Then $\frac{1}{2}$-IMB holds if and only if $\theta_1 \leq \theta_2 + \frac{1}{2}$.

Example (Shifting demands)

For any two D_1, D_2, monotonicity is satisfied for $\{D_1(p), D_2(p) + \delta\}$ for all $\delta \in (\delta_1, \delta_2)$, $\delta_1 < \delta_2$.

Outline

1. Implications
2. Intuitions for the three conditions
   a. no exclusion
   b. the expression for two demands
   c. the separability condition
3. Examples and applications

Necessity of no exclusion: $p^*(\theta) \leq \bar{p}(\theta')$ for all $\theta, \theta' \in \Theta$

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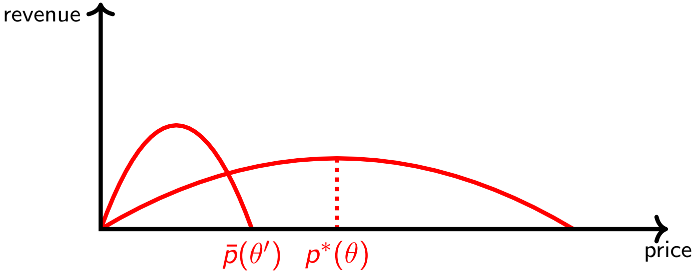
Similar to BBM'15, Pram'21.

Suppose $p^*(\theta) > \bar{p}(\theta')$. Show IMB is violated:

- Consider $s \ni \mu$ that puts almost all mass on θ, some mass on θ'.
- θ' will be "excluded" in μ.
- Separating some θ' consumers is an improvement.



Folklore: Price discrimination can be good “when it opens new markets”

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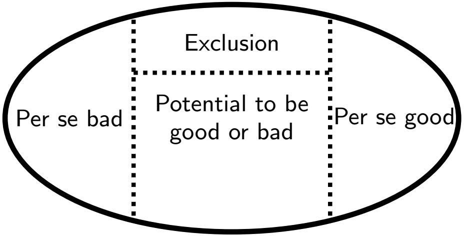

The monotonicity condition for $\{D_1, D_2\}$: the proof

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For $\mu \in [0,1]$, define $p(\mu), W(\mu)$

$$0 = (1-\mu)R_1'(p(\mu)) + \mu R_2'(p(\mu))$$
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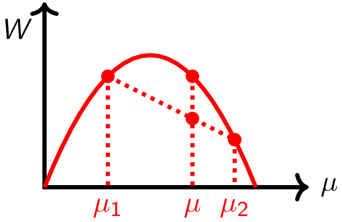

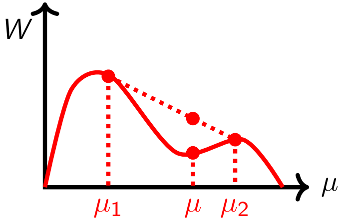

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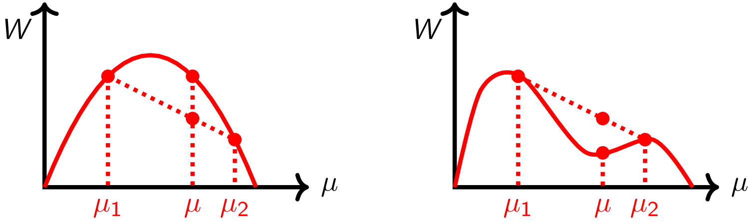
IMB $\Leftrightarrow$ W is concave. So IMB if and only if W' is decreasing,

$$W'(\mu) = V_2(p(\mu)) - V_1(p(\mu)) + \mathbb{E}[V_i'(p(\mu))]p'(\mu)$$
$$V_2(p) - V_1(p) + \frac{-\frac{R_1'(p)}{R_2'(p)}V_2' + V_1'}{-\frac{R_1'(p)}{R_2'(p)}R_2'' + R_1''}(R_1'(p) - R_2'(p))$$



The monotonicity condition for $\{D_1, D_2\}$: the three effects of information

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\begin{aligned}
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& +2p'(\mu) \quad \Big[V_2'(p(\mu)) - V_1'(p(\mu)) \Big] \\
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Split $(\frac{1}{2}, \frac{1}{2})$ into $(\frac{2}{3}, \frac{1}{3})$ and $(\frac{1}{3}, \frac{2}{3})$ with prices $p_1 < p_2$:

The monotonicity condition for $\{D_1, D_2\}$: the three effects of information

		p_1	p	p_2
Before	Type 1	0	1	0
	Type 2	0	1	0
After	Type 1	$\frac{2}{3}$	0	$\frac{1}{3}$
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example comparison

The separability condition

Starting point: Strong duality of Kolotilin 2018 and Kolotilin, Corrao, Wolitzky 2024.

Proposition

Let G be distribution over (θ, p). G is optimal if and only if there exists continuous functions $\lambda(\theta), \zeta(p)$ such that

$$\lambda(\theta) + \zeta(p) R_p(p, \theta) = U(p, \theta), G\text{-almost surely}$$
$$\lambda(\theta) + \zeta(p) R_p(p, \theta) \geq U(p, \theta), \forall (p, \theta) \in I \times \Theta$$

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1. Consider μ_0, p. No-info optimal iff $\exists \zeta, \forall \theta, \quad p \in \arg\max_{p'} U(p', \theta) - \zeta(p') R_p(p', \theta)$.
2. IMB iff no-info optimal <u>for all</u> μ_0: $\exists \zeta, \forall p, \theta, p \in \arg\max_{p'} U(p', \theta) - \zeta(p, p') R_p(p', \theta)$.

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One direction: suppose $D(\theta) = f_1(\theta) D(\theta_1) + f_2(\theta) D(\theta_2)$ and $\{D(\theta_1), D(\theta_2)\}$ IMB:

1. $\exists \zeta, \forall p, \theta \in \{\theta_1, \theta_2\}, p \in \arg\max_{p'} U(p', \theta) - \zeta(p, p') R_p(p', \theta)$.
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Outline

1. Implications
2. Intuitions for the three conditions
3. Examples and applications

Example 1: The class $aD + b$.

Proposition

α-IMB (α-IMG) holds for $\mathcal{D} = \Big\{a(\theta)D(p) + b(\theta)\Big\}_\theta$ *if and only if*

$$(2\alpha - 1)p + \alpha(\frac{pD'(p)}{R''(p)})$$

is increasing (decreasing) over $[\min_\theta p^*(\theta), \max_\theta p^*(\theta)]$.

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1. 0.5-IMB holds if and only if $p^2 f(p)$ is log-concave.
   1. Sufficient: f is log-concave, e.g., uniform $f(p) = c$ (generalizing Pigou's observation).
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Four cases for how information affects CS and TS.

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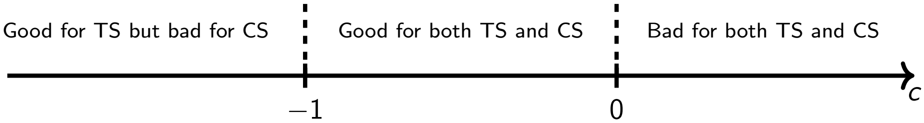
~~Four~~ Three cases for how information affects CS and TS. Consider $f(p) = \frac{(1+cp)^c}{p^2}$



Related Literature

Full vs. no segmentation, focus on either TS or CS:

- Pigou 1920; Robinson 1933; Varian 1985; Aguirre, Cowan, Vickers 2010; ...
- "Output" and "misallocation" effects are related to our three effects comparison

All segmentations based on values:

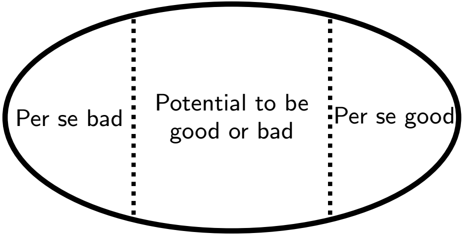
- Bergemann, Brooks, Morris 2014

Duality approaches in persuasion:

- Kolotilin 2018; Dworczak, Martini 2019; Kolotilin, Corrao, Wolitzky 2023; Smolin, Yamashita 2023; Dworczak, Kolotilin 2023

Conclusions

A characterization of:



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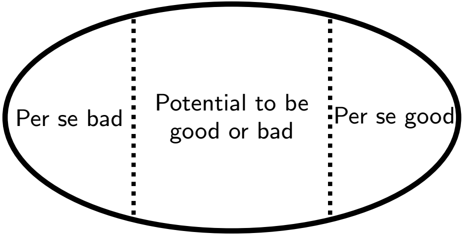

More in the paper: Approaching unit demands here, More examples here

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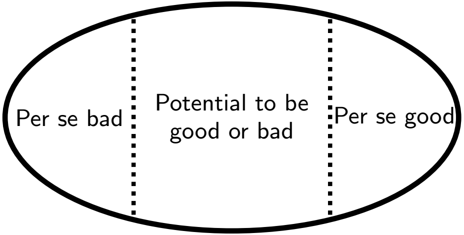

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Methodologically:

- We apply modern frameworks (endogenous segmentation) and tools (concavification and duality) to study a classical problem

Thanks!

Connections to output and misallocation effects

Price discrimination has two effects

1. Output effect
2. Missallocation effect (which is bad for TS)

the three effects literature

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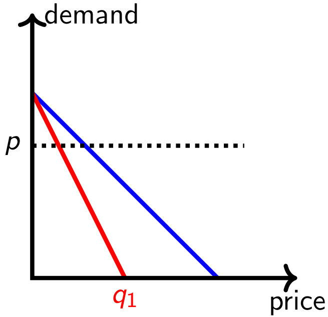

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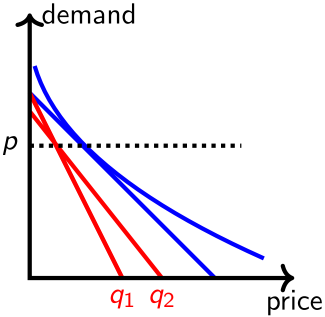
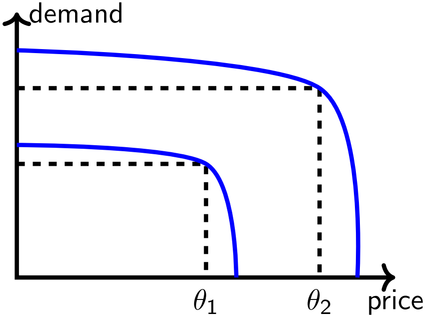


Example 2: Binary class $(\theta_1 - p)^c, (\theta_2 - p)^c$, $0 \leq \theta_1 < \theta_2$ and $c \in (0, 1]$.



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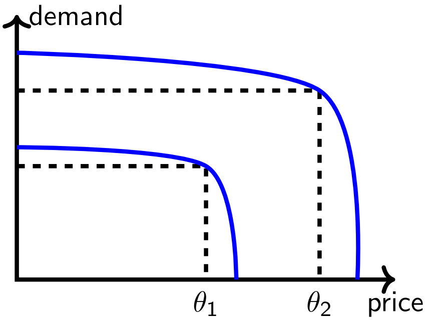
- E.g., $c = 1$: linear demands.



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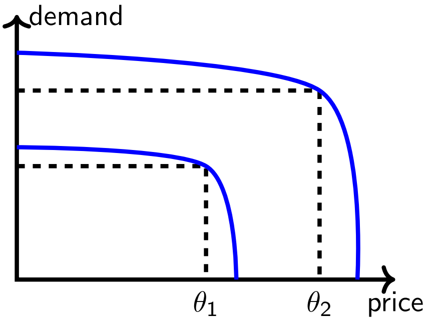
- Information remains monotonically bad if and only if there is no exclusion



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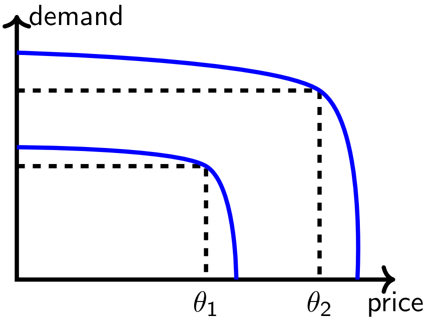

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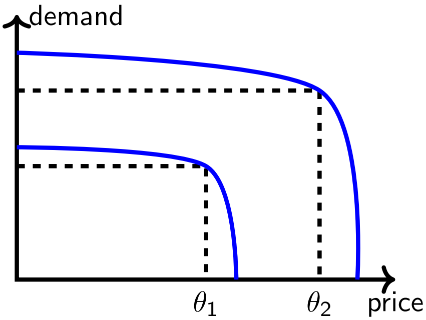
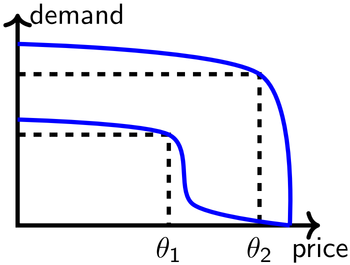
- Information remains monotonically bad if and only if there is no exclusion
  - No exclusion is violated as $c \to 0$ because $\frac{\theta_2}{1+c} > \theta_1$, so IMB doesn't hold.
- BBM: in the limit, there is some information that benefits consumers.



What if we approach unit-demand curves without violating no exclusion?



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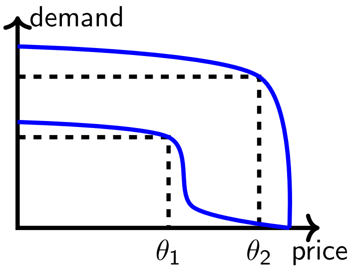
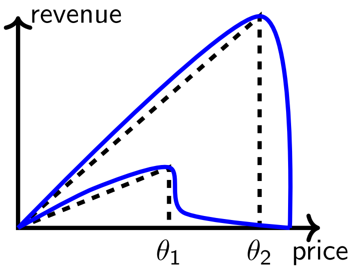
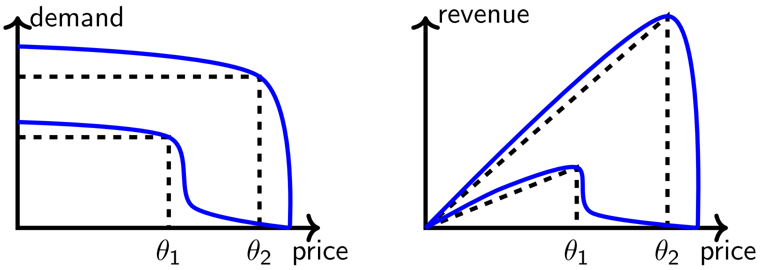




What if we approach unit-demand curves without violating no exclusion?

Corollary

Consider $\mathcal{D}^\epsilon$ that uniformly converges to a family of unit-demand curves as $\epsilon \to 0$ and revenue is concave for every $\epsilon > 0$. For small enough ϵ, the partial-inclusion condition is violated and therefore information is neither monotonically good nor bad.



Example 3: The three effects have same sign $\Rightarrow$ monotonicity

Example (Sufficient Condition for α-IMG and α-IMB)

Consider two demand curves $D(p, \theta_i) = a_i - p + \frac{c_i}{p}$ for $i \in \{1, 2\}$ and $a_i, c_i \geq 0$. Without loss of generality assume $a_1 \leq a_2$. Then α-IMG holds for all α if

$$c_1 - c_2 \geq (a_2 - a_1)\frac{a_2}{2}.$$

α-IMB holds for all $\alpha \geq \frac{1}{2}$ if

$$c_1 \leq c_2 \leq \frac{a_1^2}{4}.$$

Back to the three effects